

# QUARKS, GLUONS, AND LATTICES

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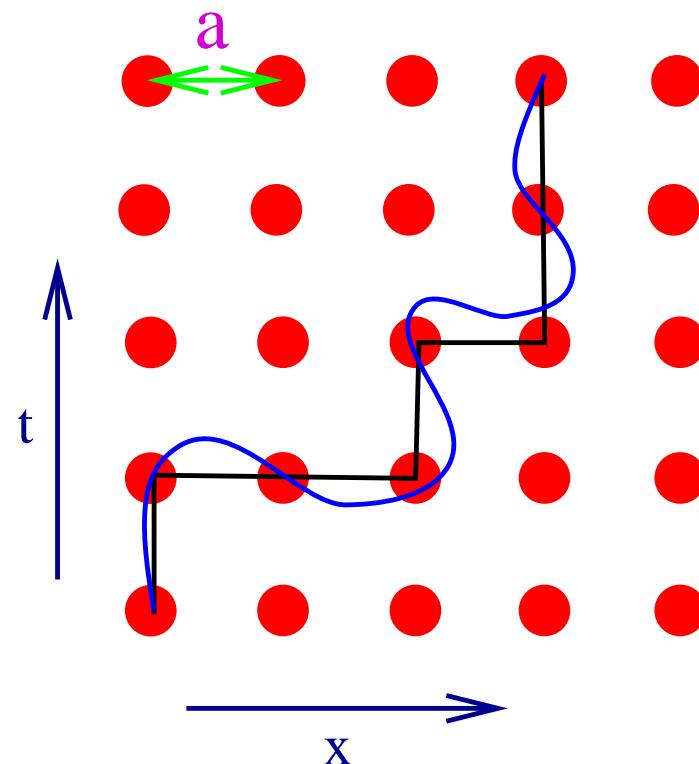


Why the lattice?

What drove us to it?

## Space-time Lattice

Quark world lines —> discrete hops



A mathematical trick

Lattice spacing  $a \rightarrow 0$  for physics

$a = \text{cutoff} = \pi/\Lambda$

## What led us to the lattice?

Late 60's

- Quantum electrodynamics: immensely successful, but “done”
- eightfold way: “quarks” explain particle spectra
- electroweak theory emerging
- electron-proton scattering: “partons”

Meson-nucleon theory failing

- $\frac{g^2}{4\pi} \sim 15$       vs       $\frac{e^2}{4\pi} \sim \frac{1}{137}$
- no small parameter

S-matrix theory

- particles are bound states of themselves
  - $p + \pi \leftrightarrow \Delta$
  - $\Delta + \pi \leftrightarrow p$
- roots of duality  $\longrightarrow$  string theory
- What is elementary?

## Early 70's

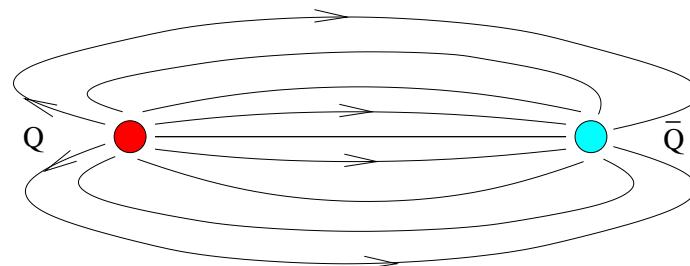
- “partons”  $\longleftrightarrow$  “quarks”
- renormalizability of non-Abelian theories
- asymptotic freedom
- Quark Confining Dynamics (QCD) evolving

## Confinement?

- interacting hadrons vs. quarks and gluons
- What is elementary?

## Flux tubes:

- $E \sim Kr$  at long distances



- $K = 14$  tons

Mid 70's

- particle theory revolution
- $J/\psi$
- quarks inescapable
- field theory reborn
- “standard model” evolves

Extended objects

- “classical lumps” a new way to get particles
- “bosonization” very different formulations can be equivalent
- growing connections with statistical mechanics
- What is elementary?

Field Theory >> Feynman Diagrams

Field theory has divergences

- bare charge, mass divergent
- must “regulate” for calculation
- Pauli Villars, dimensional regularization: perturbative
- based on Feynman diagrams

But important non-perturbative effects

- confinement
- chiral symmetry breaking

need a “non-perturbative” regulator

Wilson’s strong coupling lattice theory (1973)

- strong coupling limit confines
- only hadrons can move

space-time lattice = non-perturbative cutoff

## Lattice gauge theory

- A mathematical trick
- Minimum wavelength = lattice spacing  $a$
- Maximum momentum =  $\pi/a$
- Allows computations
- Defines a field theory

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## Lattice gauge theory

- A mathematical trick
  - Minimum wavelength = lattice spacing  $a$
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  - Allows computations
  - Defines a field theory
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- Be discrete, do it on the lattice
  - Be indiscreet, do it continuously

# What is a gauge theory

Many answers:

## 1. generalize electromagnetism

- one photon  $\longrightarrow$  several “gluons”
  - $F_{\mu\nu} \longrightarrow F_{\mu,\nu}^\alpha$
  - symmetric between “colors”
- fields charged with respect to each other

## 2. a local symmetry

- invariance under “rotations” between gluon species
- independent rotation at each space-time point
  - $g(x_\mu)$  arbitrary function of space time
- gravity analogue: local coordinates

## 3. fields not simply Lorentz invariant

- gauge rotations under a Lorentz transformation
- gravity analogue: Christoffel symbols

## 4. A theory of phases

- a static particle in electric potential:  $E \rightarrow E + gA_0$ 
  - $\psi \sim e^{iEt} \rightarrow e^{igA_0 t} \psi$
- generalizes to a moving particle

$$\psi \rightarrow \exp(ig \int_x^y A^\mu dx_\mu) \psi$$

- Non-Abelian theory:  
phase  $\rightarrow$  matrix
- the basis of the lattice formulation

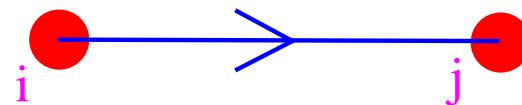
## Wilson's formulation

local symmetry + theory of phases

Variables:

- Gauge fields generalize “phases”

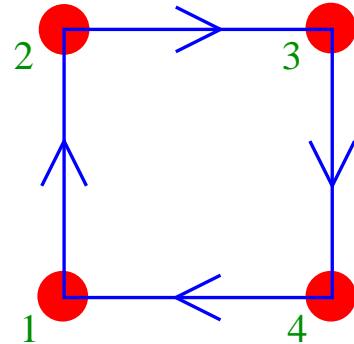
$$U_{i,j} \sim \exp(i \int_{x_i}^{x_j} A^\mu dx_\mu)$$



- On links connecting nearest neighbors
- $U_{ij}$  = 3 by 3 unitary matrix  $\in \text{SU}(3)$
- 3 quarks in a proton

Dynamics:

- Sum over elementary squares, “plaquettes”



$$U_p = U_{1,2}U_{2,3}U_{3,4}U_{4,1}$$

- like a “curl”
- flux through corresponding plaquette.

$$S = \int d^4x F^{\mu\nu} F_{\mu\nu} \longrightarrow \sum_p \left( 1 - \frac{1}{3} \text{ReTr} U_p \right)$$

Quantum mechanics:

- via path integral
- sum over paths —> sum over phases

$$Z = \int (dU) e^{-\beta S}$$

- invariant group measure
- $\beta$  defines the “bare” charge

$$\beta = \frac{6}{g_0^2}$$

- must renormalize as  $a \rightarrow 0$

## Parameters

$$a \rightarrow 0$$

Asymptotic freedom (2004 Nobel prize!):

$$g_0^2 \sim \frac{1}{\log(1/a\Lambda)} \rightarrow 0$$

Overall scale  $\Lambda$  from “dimensional transmutation”

- Coleman and Weinberg
- depends on units: not a real parameter

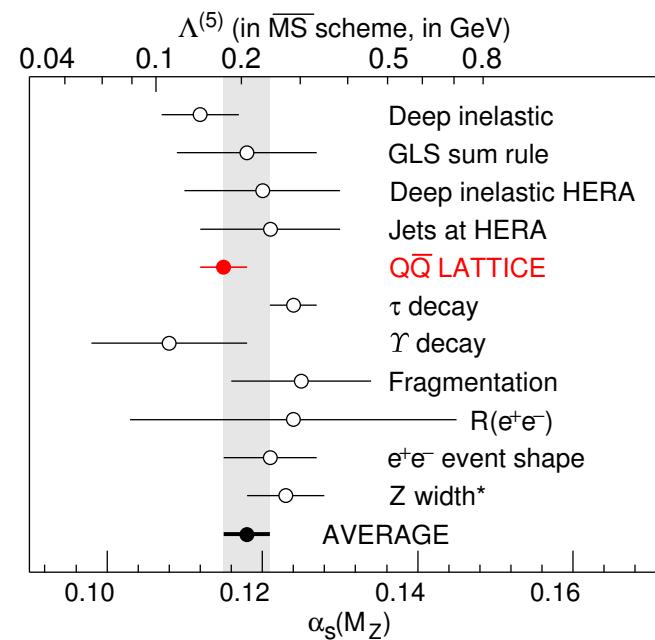
Only the quark masses!

$m_q = 0$ : parameter free theory

- $m_\pi = 0$
- $m_\rho/m_p$  determined
- close to reality

## Example: strong coupling determined

$$\alpha_s(M_Z) = 0.115 \pm 0.003$$



(PDG, 1999)

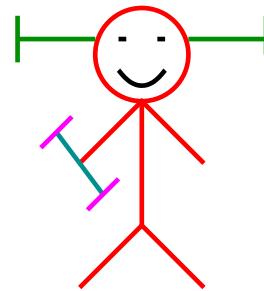
(charmonium spectrum for input)

## Monte Carlo methods

Make random field changes biased by Boltzmann weight.  
Converge towards configurations in “thermal equilibrium.”

$$P(C) \sim e^{-\beta S}$$

In principle can measure anything.  
Fluctuations → theorists have error bars!

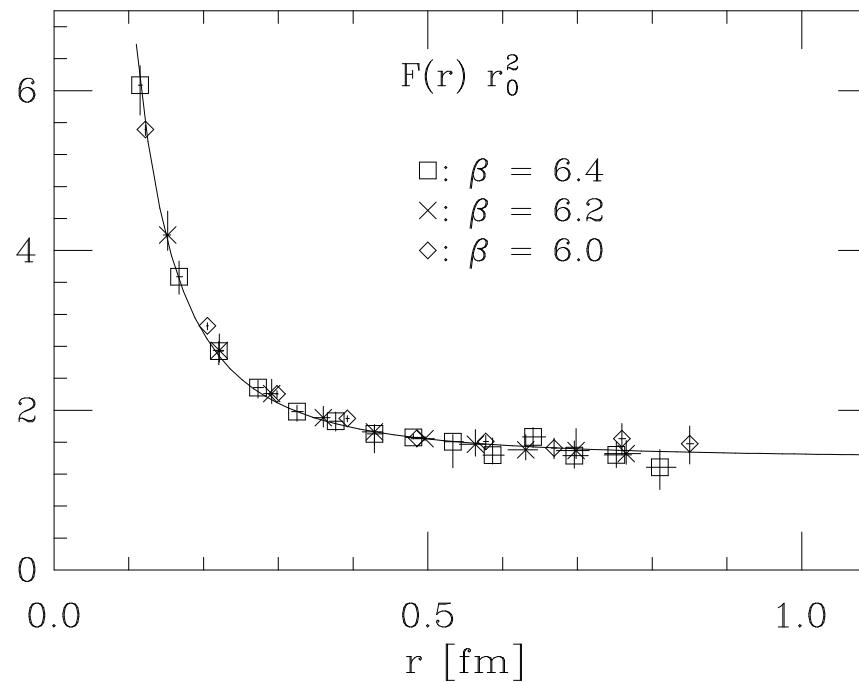


Systematic errors:

- finite volume
- finite lattice spacing
- quark mass extrapolations

## Interquark force

- constant at large distance
- confinement



C. Michael, hep-lat/9509090

Hadronic spectra: as  $t \rightarrow \infty$

$$\langle \phi(t) \phi(0) \rangle \longrightarrow e^{-mt}$$

- $m$  = mass of lightest hadron created by  $\phi$
- Bare quark mass is a parameter

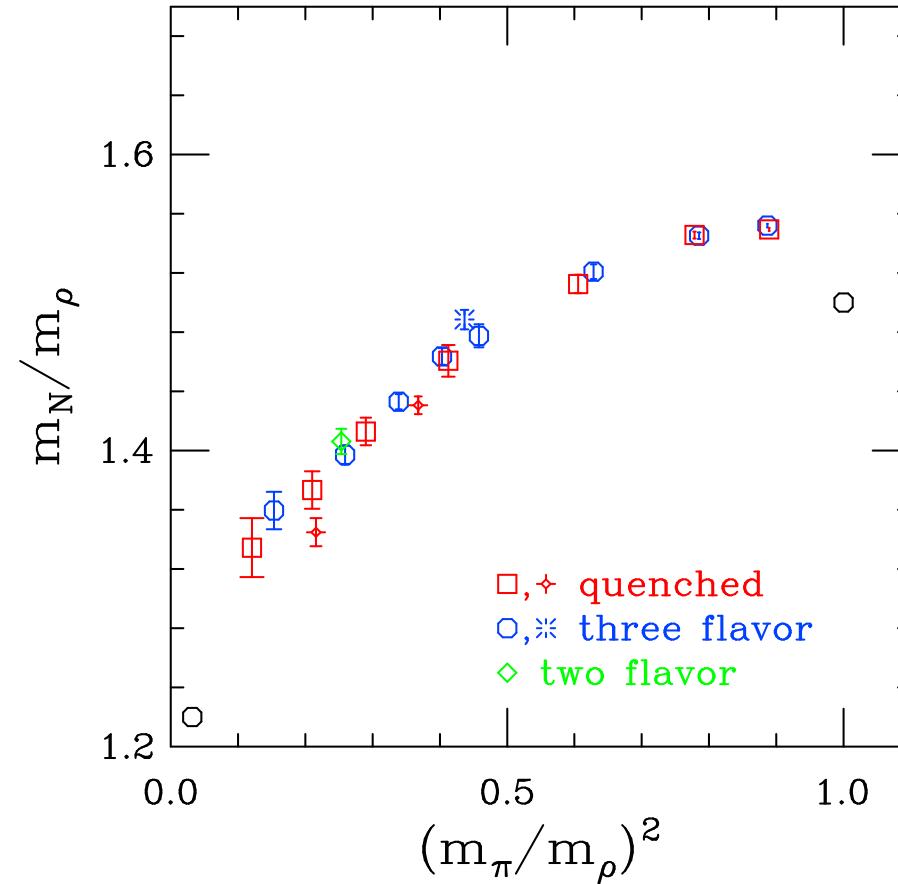
Chiral symmetry:

$$m_\pi^2 \sim m_q$$

Adjust  $M_q$  to get  $m_\pi/m_\rho$  ( $M_s$  for the kaon)

all other mass ratios determined

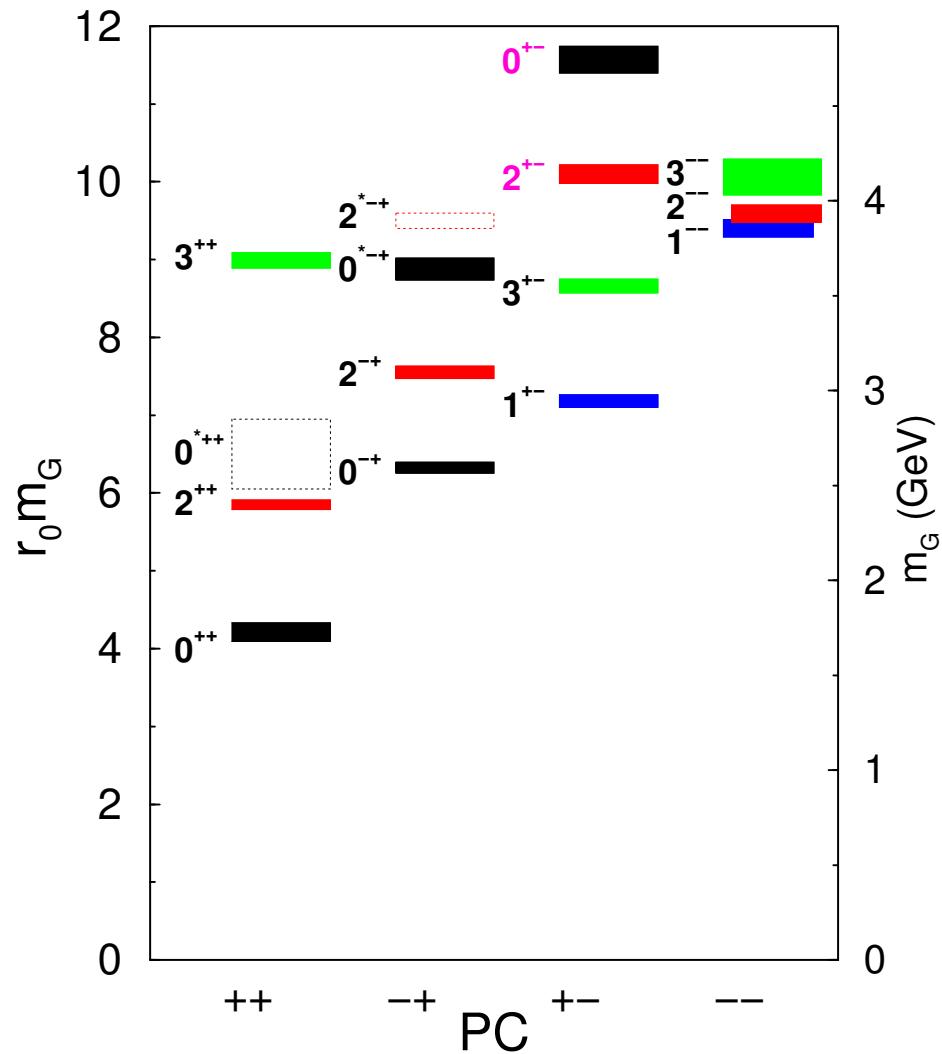
“APE” or “Edinburgh” plot:



- improved Kogut-Susskind quarks,  $16^3 \times 48$  lattice
- MILC collaboration, Phys. Rev. D 64, 054506 (2001)

## Glueballs

- gluonic excitations
- no quarks



- Morningstar and Peardon, Phys. Rev. D 60, 034509 (1999)
- quenched, anisotropic lattice

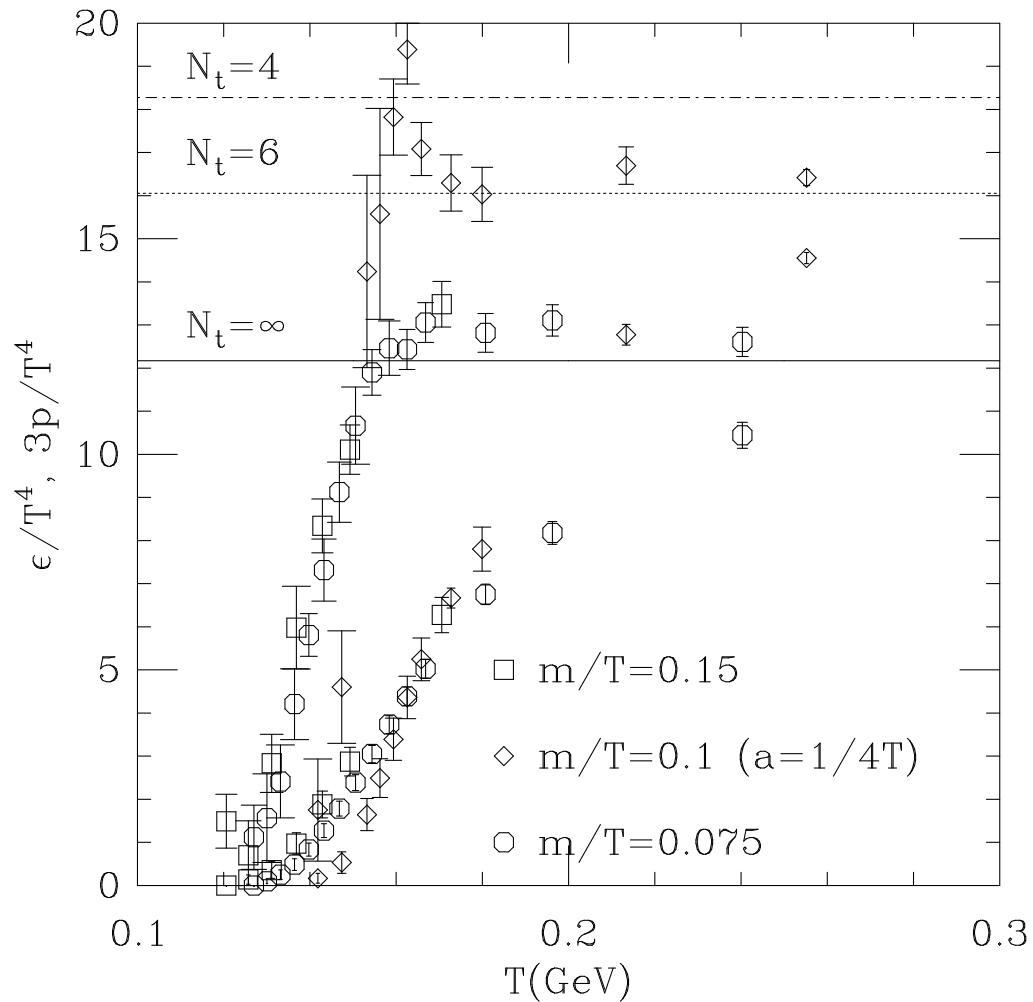
## Quark Gluon Plasma

Finite temporal box of length  $t$

$$Z \sim \text{Tr } e^{-Ht}$$

- $1/t \leftrightarrow$  temperature
- confinement lost at high temperature
- chiral symmetry manifestly restored
- $T_c \sim 235$  MeV, 0 flavors (quenched)
- $T_c \sim 170$  MeV, 2 flavors

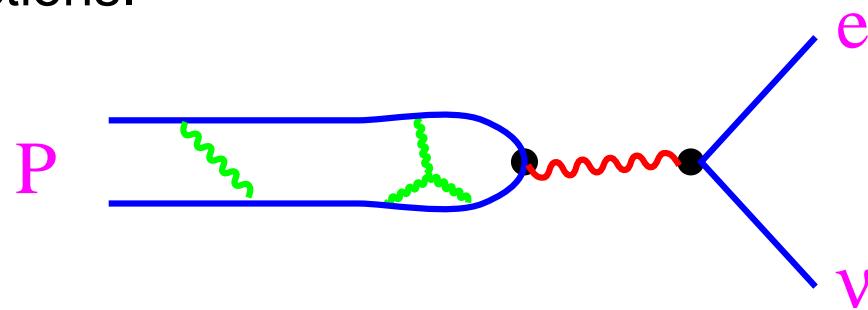
Energy  $\epsilon$  and pressure  $p$  versus temperature.



Bernard *et al.*, MILC collaboration, Dec. 1996

## Matrix elements

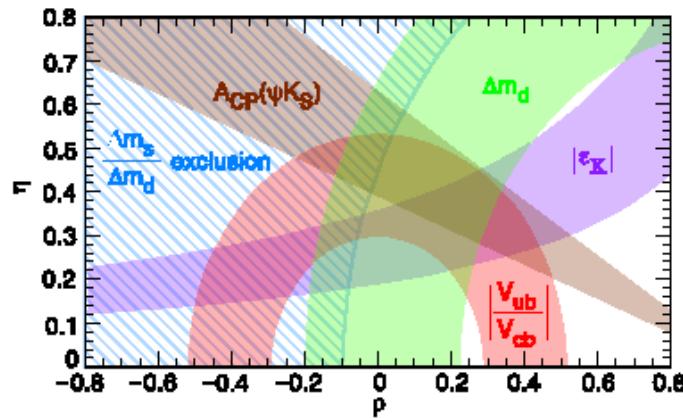
To test standard model predictions for weak decays, need strong interaction corrections.



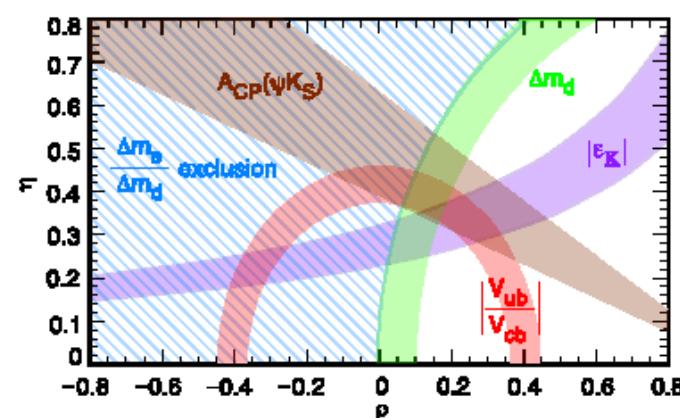
- $\Delta I = 1/2$  rule verified
- $\epsilon'/\epsilon$  large quenching errors, heavy use of chiral perturbation theory
- dynamical simulations necessary: QCDOC

# Impact of Reduced Lattice Errors

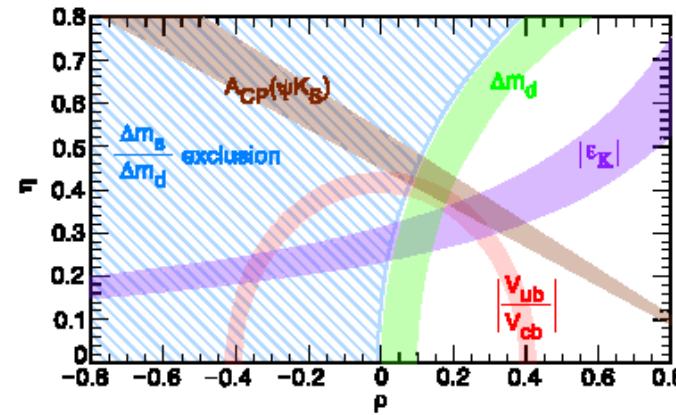
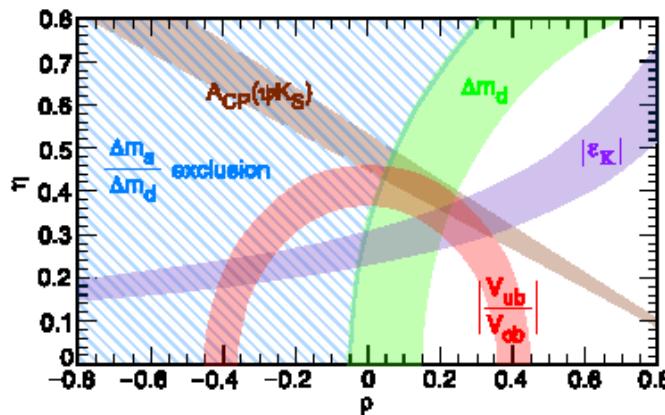
CKM today ...



... and with 2–3% theory errors.



And with B Factories ...



The impact of the B factories and improvements in lattice calculations on parameters of the CKM matrix. CLEO-c Collaboration (2001).

# Quarks: serious unsolved problems

## Anticommuting fields

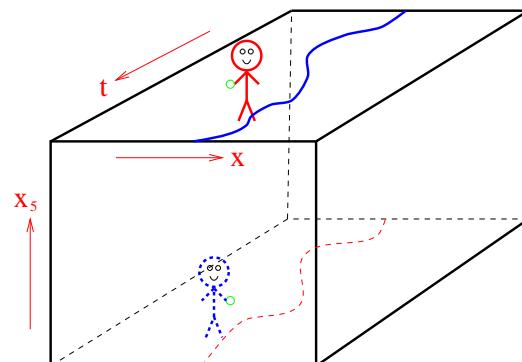
- $\not\equiv$  classical statistical mechanics
- Integrate out as a determinant
- Tedious to simulate.

## Chemical potential background baryon density

- Non-positive weight.
- No viable algorithms known!

## Chiral fermions and the “standard model”

- Unsolved difficulties tied with anomalies.
- Lots of recent activity.
- My favorite: 4d world an interface in 5d



## The Lattice SciDAC Project

66 US lattice theorists; 9 member executive committee:

R. Brower, (Boston U.) N. Christ (Columbia U.), M. Creutz (BNL), P. Mackenzie (Fermilab), J. Negele (MIT), C. Rebbi (Boston U.), D. Richards (JLAB), S. Sharpe (U. Washington), R. Sugar (UCSB)

Two prong approach

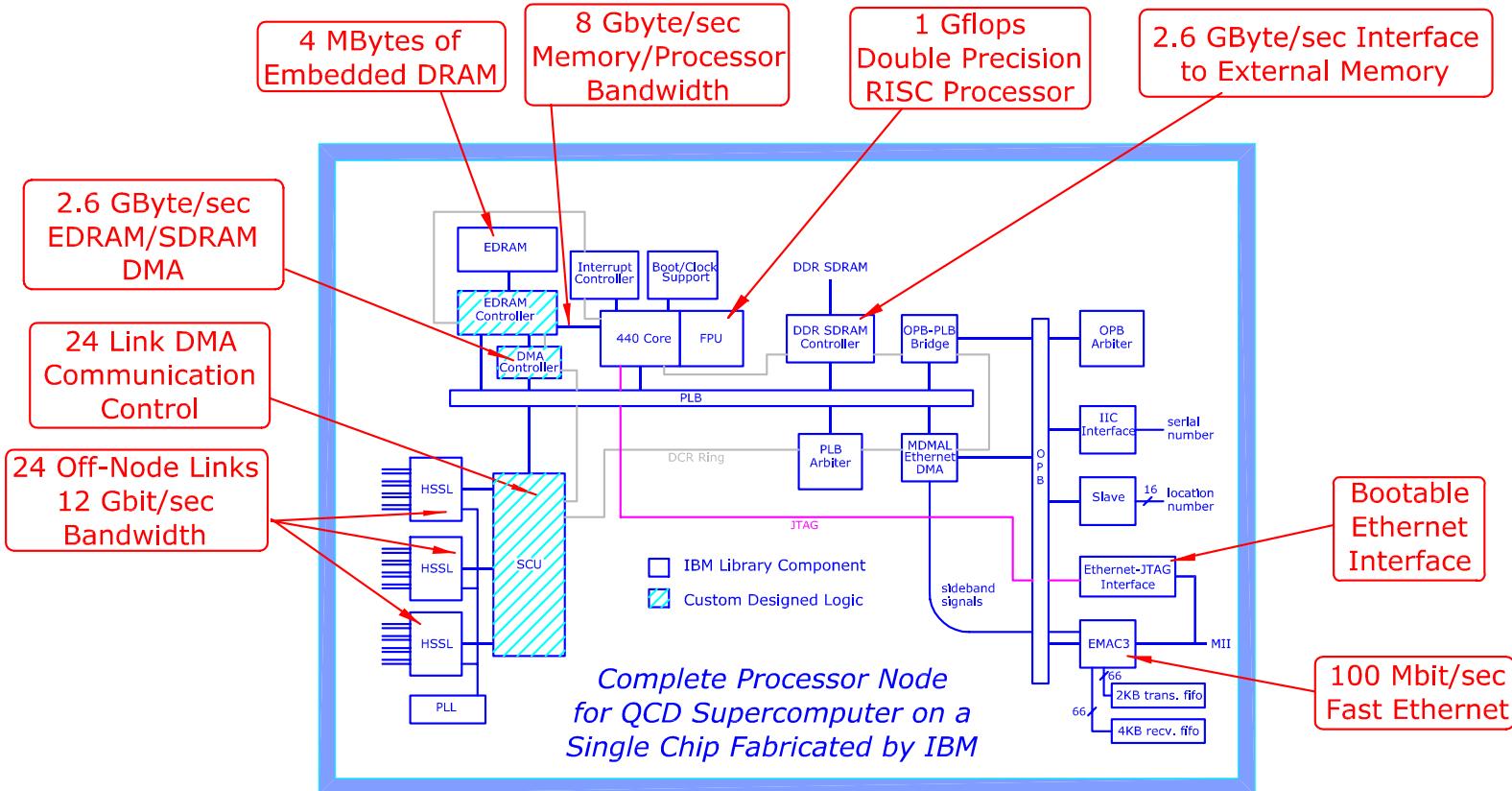
- QCDOC at BNL
- commodity clusters at Fermi Lab and Jefferson Lab
- $\sim 3 \times 10$  Teraflops distributed computing facility

QCDOC

- next generation after QCDSP
- designed by Columbia University with IBM
- on design path to IBM Blue Gene
- Power PC nodes connected in a 6 dimensional torus
- processor/memory/communication on a single chip

QCDOC places entire node on a single custom chip

## QCDOC ASIC DESIGN



Mission-critical, custom logic (hatched) for high-performance memory access and fast, low-latency off-node communications is combined with standards-based, highly integrated commercial library components.



Two node daughterboard



64 node motherboard



128 node prototype



DOE/RIKEN 24,576 nodes!



128 node dual 2.4GHz P4 Myrinet cluster, commissioned at FNAL in January 2003



256 node single 2.66 GHz P4 Gigabit Ethernet cluster, commissioned at JLab in September 2003

# Unsolved Problems

## Chiral gauge theories

- parity conserving theories in good shape
- chiral theories (neutrinos) remain enigmatic
- non-perturbative definition of the weak interactions?
- related problem: supersymmetry

## Sign problems

- chemical potential
  - color superconductivity
  - doped semiconductors
- $\theta \neq 0$ 
  - spontaneous CP violation at  $\theta = \pi$

## Fermion algorithms

- remain very awkward
- why treat fermions and bosons so differently?

We need new ideas!